Evaluation of measurement uncertainty in permanent deformation tests of lateric soils

Determinação da incerteza de medição nos ensaios de deformação permanente de solos laterícos

Juliana Tanabe Assad dos Santos¹, Paulo Afonso Lopes da Silva², Antonio Carlos Rodrigues Guimarães³

¹Military Institute of Engineering, Rio de Janeiro – Brazil, juliana.tanabe@ime.eb.br
²Military Institute of Engineering, Rio de Janeiro – Brazil, pauloafonsolopes@uol.com.br
³Military Institute of Engineering, Rio de Janeiro – Brazil, guimaraes@ime.eb.br

ABSTRACT

During a process of laboratory analysis of soils, several variables may influence the determination of the aimed parameter. Inherently, each of the variables has associated uncertainties, which may be related to factors such as calibration, data reading, malpractice and so on. This article aims to determine the uncertainty of measurements of permanent deformations in lateritic soils used for pavement purpose. Experimental data were obtained from a repeated load triaxial test on clayey lateric sand in Mimoso do Sul, state of Espírito Santo. The methodology includes the uncertainties associated with the confining pressure (σ₃), the deviator stress (σ₄) and the number of cycles, generally informed by a calibration certificate. The samples were divided into two groups, differentiated only by the value of σ₄ applied. The results of the test were expressed by coverage intervals, which consider expanded uncertainties, with a coverage probability of 95%; comparing the results, the biggest contribution to the combined uncertainty is the confining stress. It was observed that the measurement uncertainties present values with practical significance, which allows the use in pavements, and that their determinations guarantee the quality of the results, because every measurement has an error associated with it, and without a quantitative determination, the measurement has no value.

RESUMO

Durante um processo de análise laboratorial de solos, diversas variáveis influenciam na determinação do parâmetro pretendido. Inerentemente, cada uma das variáveis apresenta incertezas associadas, que podem estar relacionadas a fatores como calibração, leitura de dados, imperícia, etc. Diante do exposto, o presente artigo tem como objetivo determinar a incerteza das medições das deformações permanentes em solos lateríticos. Foram utilizados dados experimentais obtidos de ensaio triaxial de cargas repetidas realizados em areia argilosa do município de Mimoso do Sul, estado do Espírito Santo. A metodologia utilizada engloba as incertezas associadas à tensão confinante (σ₃), à tensão de desvio (σ₄) e ao número de ciclos (N), em geral informadas por certificado de calibração. As amostras foram divididas em dois grupos, diferenciados apenas pelo valor de σ₄ aplicado. Os resultados das medições de dois grupos de ensaio são expressos em forma de intervalo que inclui a incerteza calculada. Como resultado, pode-se observar que a incerteza de medição para o caso proposto não representa variação significativa do resultado para aplicação em pavimentos. Além disso, destaca-se a grande influência da tensão confinante na avaliação desse parâmetro ao comparar os resultados dos dois grupos. Concluiu-se que apresentar a incerteza das medições evidencia a confiabilidade e qualidade dos resultados.

Keywords:
Stascal analysis.
Tropical soils.
Mechanical behavior.

Palavras-chave:
Análise estatística.
Solos tropicais.
Comportamento mecânico.

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1. INTRODUCTION

Tropical soils are soils that can be found in tropical climate environments that present peculiar behaviors as a result of geological and pedological processes associated with this type of climate. These materials are found in many areas of the Brazilian territory (Nogami and Villibor, 2007).

Lateritic soils are one of the three classifications that constitute the group of tropical soils, found abundantly in the country, except in areas that have not undergone a laterization process (Nogami and Villibor, 1991). These soils used to be discarded in road pavement process, due to incompatibility with Brazilian standards, which are based on international standards.

However, in light of new studies and technologies, and from the dissemination of mechanistic method for materials evaluation, these soils have been taken into consideration as components of road pavements layers. Soil analysis by the mechanistic method is centered on repeated load triaxial test. Based on this test, the elastic and plastic behavior are analyzed and expressed, respectively, by the resilient modulus (RM) and by the permanent deformation (PD).

Nevertheless, it is understood that the results obtained in laboratory tests are influenced by several sources of uncertainty, such as equipment calibration, measurement readings, etc. Uncertainty determinations guarantee the quality of the results, because every measurement has an error associated with it, and without a quantitative determination of that error, the measurement is worthless.

Thus, in order to understand the measurements imprecision impact and evaluate test results quality, this work determined the uncertainties of the measurements of lateritic soils applied in road subgrade layer, based on the ABNT ISO/IEC GUIA 98-3 (2014), based on data from tests carried out by Guimarães (2009) in clayey soil with lateritic characteristics, in the region of Mimoso do Sul, ES.

2. BIBLIOGRAPHIC REVIEW

Lateritic soils have an exclusive genesis of places with a tropical environment, which directly influences materials properties (Kamtchueng et al., 2015; Camapum de Carvalho et al., 2015). Among the peculiarities associated with the soil laterization process, there can be mentioned the presence of clay-mineral kaolinite and the enrichment of hydrated oxides of iron and aluminum (Nogami and Villibor, 2007).

Due to the significant occurrence in Brazil, lateritic soils have become a viable option for road pavement layers composition. It is observed that, despite having low values of California Bearing Ratio (CBR) under natural conditions, lateritic soils present good mechanical behavior when applied at optimal moisture content (Nogami and Villibor, 2007). These soils have been used mainly as base layer, subbase and subgrade in asphalt pavements, but they can also be found as prime coat in several access roads (Guimarães, Silva Filho and Castro, 2021; Guimarães, 2009).

One of the most common non-conformities related to these pavement layers is permanent deformation (Bernucci et al., 2008), also known as wheel track or rutting. This pathology consists of accumulation of small non-recoverable displacements in response to traffic (Motta, 1991; Guimarães, 2001), quantified by the specific permanent deformation value.

In laboratory, permanent deformation is determined through repeated load triaxial equipment, whose test consists of the application of repeated load cycles load for a certain
stress state in specimens. These procedures are based on the DNIT 179/2018 - IE. The analysis of permanent deformation through this test appears in recent studies: Guimarães (2018), Lima et al. (2020), Silva et al. (2021) and Lima and Motta and Aragão (2021), Bona and Guimarães (2021) and Aragão et al. (2019).

Another test considered effective for PD tests is the multi-stage repeated load triaxial test, which allows the evaluation of several stress pairs from only one specimen, unlike the standard repeated load triaxial test. The results obtained by this test, already standardized by the European Committee for Standardization (ECS), allow an efficient evaluation of permanent deformation in granular materials, as pointed out by Cabral (2021) and Erlingsson and Li et al. (2019).

Several studies showed low values of permanent deformation in lateritic sandy soils (LA'): those soils studied by Silva et al. (2021) resulted in a maximum deformation of 0.22%; seven soils of the same classification tested by Sousa (2021) presented varied permanent deformations, the lowest value being lower than 0.5% and the highest been approximately 3%. Similar results were obtained by Lima (2020) and by Lima et al. (2020).

Concerning lateritic clays (LA), Lima et al. (2021) observed, through two samples, PD values between 1% and 2%, while permanent deformations lower than 1% were obtained for lateritic clayey soil (LG') by Lima et al. (2020). For soil samples LG', Sousa (2021), Lima (2020) and by Lima et al. (2020) observed permanent deformations between 0.6% and 3.6%.

Table 1 presents some of these soils permanent deformations associated with the applied stresses for the number of applied cycles (N) of 150,000. The other soils could not be presented in this table due to incompatibility of stresses, number of cycles or even lack of precise data in the consulted references.

<table>
<thead>
<tr>
<th>Classification</th>
<th>LG'</th>
<th>LG'</th>
<th>LA'</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>-</td>
<td>0.60%</td>
<td>0.42%</td>
</tr>
<tr>
<td>80</td>
<td>-</td>
<td>2.97%</td>
<td>1.88%</td>
</tr>
<tr>
<td>120</td>
<td>-</td>
<td>4.38%</td>
<td>4.35%</td>
</tr>
<tr>
<td>80</td>
<td>0.94%</td>
<td>2.60%</td>
<td>1.61%</td>
</tr>
<tr>
<td>160</td>
<td>0.96%</td>
<td>3.84%</td>
<td>4.76%</td>
</tr>
<tr>
<td>240</td>
<td>0.89%</td>
<td>9.00%</td>
<td>9.64%</td>
</tr>
<tr>
<td>120</td>
<td>-</td>
<td>4.85%</td>
<td>2.97%</td>
</tr>
<tr>
<td>240</td>
<td>-</td>
<td>3.83%</td>
<td>6.51%</td>
</tr>
<tr>
<td>360</td>
<td>-</td>
<td>7.56%</td>
<td>13.33%</td>
</tr>
</tbody>
</table>

The uniformity conditions of the material and the importance given to the study at the time of the test are factors that influence the determination of permanent deformation (Bishop and Henkel, 1962). For these reasons, the large number of variables in laboratory tests make it difficult to obtain accurate and homogeneous results.

In this context, EN ISO/IEC 17025 (2017) determines that laboratories must apply procedures to estimate measurement uncertainties in order to guarantee the reliability of measurement results obtained in laboratories.

Measurement uncertainty is the parameter associated with the result of a measurement, which characterizes values dispersion that can reasonably be attributed to the measurand (quantity to be measured). For its estimation, the sources of uncertainty and their contributions to the final uncertainty are considered.
In transportation engineering, measurement uncertainty has been considered in materials area. Lopes (2014), for example, determined this estimate in bituminous mixtures tests through permanent deformation test analysis. The author concluded that the equipment and the preparation and packaging processes, and equipment are sources of uncertainty for the determination of the final uncertainty.

Carrasco, Carvalho and Oliveira (2008) presented a calculation methodology for measuring the reliability of measurements in compression tests parallel to the fibers, determining the combined uncertainty, whose value indicated high quality of equipment and its calibrations.

A study of uncertainties propagation in soil compaction tests was carried out by Jesus et al. (2016), and it was verified that the adjustments to the ABNT NBR 6457:2016 and ABNT NBR 7182:2020 standards made it possible to reduce the standard uncertainties of moisture content and density in the tests.

Assali, Fortes and Cymrot (2003) compared the measurement uncertainties referring to the CBR and Mini CBR tests. It was observed that the Mini CBR tests presented higher uncertainties.

It was evaluated by Andrade et al. (2011) the repeatability and reproducibility for calculating uncertainty in soil characterization. In the granulometric analysis, the authors obtained uncertainties between 2% and 3%.

Reeves, Knight and Zebker (2014) presented an analysis of uncertainties associated with soil surface deformations measurement for hydraulic purposes in agricultural areas. The technique used was synthetic aperture radar interferometry, with measurement uncertainty between 0.21 cm and 0.27 cm.

In order to increase mapping accuracy on soil contamination with heavy metals, Horta et al. (2021) analyzed uncertainty measurement of portable X-Ray fluorescence (pXRF) equipment, using sequential geostatistical simulation methods, varying probabilities to assess the uncertainty of each sample. They concluded that there was a cost-effective solution for the direct use of pXRF data.

Jeong et al. (2016) analyzed the uncertainty of flux stresses in metal sheets subject to large plastic deformations, based on inaccuracies related to the X-Ray diffraction (XRD) equipment, determining the optimal condition to use the equipment and to obtain quality results.

3. METHODOLOGY

The methodology of this research is divided into three stages: 1) measurement system modeling, measurand determination and uncertainty sources identification; 2) determination of a factor associated with the confidence of the result, multiplied by a combined standard uncertainty, from the standard uncertainty of each source and its contribution to the expanded uncertainty, and 3) expanded uncertainty determination, which expresses the soil’s permanent deformation uncertainty. These steps are in the flowchart in Figure 1, and detailed in items 3.1 to 3.6.

3.1. Measurement system modeling

3.1.1. Measurement objective

To determine measurements uncertainties associated with permanent deformations in lateritic soils, from experimental data obtained from the permanent deformation test in clayey soil at normal compaction energy.
3.1.2. Materials

The material is a fine-grained lateritic soil, collected in its occurring profile, on a slope along the BR-101/ES highway. The soil is classified, according to the MCT methodology, as LG’ (clayey lateritic soil), with $c’ = 2.05$ and $e’ = 0.97$. The liquid and plastic limits are 60.3% and 22.5%, respectively, resulting in a plasticity index of 37.8%.

It is presented in Tables 2 and 3 the results of the physical-chemical analysis and the granulometric composition of the ES clayey sand. Based on the low $K_i$ value, the high degree of weathering in the material is observed, that is, this factor indicates that the soil has a low tendency of mineralogical composition in relation to the presence of expansive clay minerals.

**Table 2 – Espírito Santo clayey sand physicochemical analysis**

<table>
<thead>
<tr>
<th>pH</th>
<th>H₂O</th>
<th>KCl 1M</th>
<th>$\Delta P_%$</th>
<th>SiO₂ %</th>
<th>Al₂O₃ %</th>
<th>Fe₂O₃ %</th>
<th>TiO₂ %</th>
<th>K₂O %</th>
<th>Res %</th>
<th>Kᵢ %</th>
<th>Kr %</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.10</td>
<td>5.03</td>
<td>9.74</td>
<td></td>
<td>15.70</td>
<td>22.60</td>
<td>10.10</td>
<td>0.95</td>
<td>0.06</td>
<td>37.30</td>
<td>1.18</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Source: Adapted from Guimarães (2009)

**Table 3 – Espírito Santo clayey sand granulometric composition (%)**

<table>
<thead>
<tr>
<th>Sand</th>
<th>Clay</th>
<th>Silt</th>
<th>Fine</th>
<th>Medium</th>
<th>Coarse</th>
<th>Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>38</td>
<td>15</td>
<td>12</td>
<td>17</td>
<td>16</td>
<td>2</td>
</tr>
</tbody>
</table>

Source: Adapted from Guimarães (2009)
3.1.3. Test preparation

The soil was dried in oven at 80°C and separated into 4000 g samples inside plastic bags. After the determination of the optimal moisture content, in the value of 18%, water was added, as necessary, to reach the defined value. Finally, the materials were homogenized and separated again in plastic bags, also weighting 4000 g.

The samples were mechanically compacted at normal energy in tripartite cylinders of 10 cm in diameter and 20 cm in height. Then, the specimens were submitted to tests in repeated load triaxial equipment, belonging to COPPE/UFRJ.

The applied stresses (σ₃ and σ₄) do not meet the stresses pairs recommended by DNIT; however, the values adopted are accepted by the standard considering they are in the range of suggested stresses.

The tests were performed for different numbers of load cycles (N), most of them above 150,000 cycles, as indicated by the standard. However, in the samples analyzed in this article, permanent deformations were analyzed for 50,000 cycles.

Eight PD tests were carried out in the lateritic clayey soil and permanent deformations were measured for each sample. Taking into consideration that half of the tests were performed with a 2.14286 kgf/cm² deviation stress, and the other half with a third part of this deviation stress, tests were divided into two groups:

- Group 1 (G1) - Confining pressure and deviation stress both equal to 0.71428 kgf/cm² and N equal to 50,000 cycles, and
- Group 2 (G2) - Confining pressure equal to 0.71428 kgf/cm², deviation stress equal to 2.14286 kgf/cm² and N equal to 50,000 cycles.

3.1.4. Measurand specification and its calculation expression

The measurand corresponds to the quantity that is sought to be measured (INMETRO, 2012). For this work, the measurand is the specific permanent deformation (εₚ). From the input parameters (σ₃, σ₄ and N) and the permanent deformations obtained with experimental tests (εₚₑ), presented in Table 4, Guimarães (2009) used a multiple nonlinear regression technique through the Statistica 8.0 software to define deformability parameters (Ψᵢ) for each material. These parameters were applied in Equation 1, proposed by the author. The deformability parameters for the soil analyzed in this work are presented in Table 5.

As the PD must be calculated in percentage, and the value obtained by the test is expressed in mm, the units are regulated by multiplying the calculated value by the tested specimen height.

\[
\varepsilon_p = \Psi_1 \times (\frac{\sigma_3}{\rho_0})^{\Psi_2} (\frac{\sigma_4}{\rho_0})^{\Psi_3} N^{\Psi_4}
\]

(1)

where:
- \( \varepsilon_p \): specific permanent deformation [%];
- \( \Psi_1, \Psi_2, \Psi_3 \) e \( \Psi_4 \): model deformability parameters;
- \( \rho_0 \): tensão de referência, considerada a pressão atmosférica de 1kgf/cm²;
- \( \sigma_3 \): confining pressure [kgf/cm²];
- \( \sigma_4 \): deviation stress [kgf/cm²]; and
- \( N \): número de ciclos de aplicação de carga.

Measurand estimate is obtained by the arithmetic mean of the permanent deformations of the tests performed, Equation 2.
where: \( y \): measurand estimate [mm]; and

\[ \varepsilon_{p}^{(1)}, \varepsilon_{p}^{(2)}, \varepsilon_{p}^{(3)} \text{ and } \varepsilon_{p}^{(4)}: \text{permanent deformation of same group samples [mm].psi;} \]

**Table 4** – Permanent deformation obtained through tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen</th>
<th>( \varepsilon_{p}^{(4)} ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.612</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.623</td>
</tr>
<tr>
<td>G1</td>
<td>4</td>
<td>3.475</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.168</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.447</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>9.041</td>
</tr>
<tr>
<td>G2</td>
<td>8</td>
<td>6.273</td>
</tr>
</tbody>
</table>

Source: Adapted from Guimarães (2009)

**Table 5** – Deformability parameters

<table>
<thead>
<tr>
<th>( \Psi_{1} )</th>
<th>( \Psi_{2} )</th>
<th>( \Psi_{3} )</th>
<th>( \Psi_{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.634</td>
<td>0.093</td>
<td>1.579</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Source: Adapted from Guimarães (2009)

### 3.2. Identification of all sources of uncertainty

In order to facilitate the understanding of the sources of uncertainty that will be analyzed, an identification strategy is applied through the following steps, proposed by Lopes (2017):

**Step 1.** Write the mathematical expression (functional relationship) to estimate the value of measurand \( Y \) (Equation 3).

\[
Y = \varepsilon_{p} = \Psi_{1} \times \left( \frac{\sigma_{3}}{\rho_{0}} \right)^{\Psi_{2}} + \left( \frac{\sigma_{d}}{\rho_{0}} \right)^{\Psi_{3}} + N^{\Psi_{4}}
\]

**Step 2.** List, in bold, the parameters of the mathematical expression (functional relationship), which are the main sources of uncertainty, Level 1 of the identification scheme, \( \sigma_{3}, \sigma_{d}, \text{ and } N \).

**Step 3.** Include, still in Level 1, main sources of uncertainty that were not mentioned in Step 2. In this article, only those listed in Step 2 are considered as main sources.

**Step 4.** List, for each major source of uncertainty, the minor sources related to it. Place the text from this list further to the right to form Level 2 of the composition. In this work, both Level 1 stresses \( (\sigma_{3} \text{ and } \sigma_{d}) \) are associated with uncertainties related to the applied force and the specimen base area.

**Step 5.** List additional sources of uncertainty, related to each source already included, that constitute other levels. In this article, the area constitutes a source of additional uncertainty, since this source depends on the measured value of the circumference diameter. Thus, the diameter of the circle will compose a third level uncertainty source for the confining pressure and for the deviation stress.

**Step 6.** Complete the modeling, as shown in Table 6.

It should be noted that there are other sources of uncertainty related to the tests performed, such as the samples moisture content, soil compaction and the environment temperature at the test time. However, the present authors chose to study the three sources of uncertainties listed above (confining pressure, deviation stress and number of cycles) because they present greater control ease and data collection.
Table 6 – Final scheme for identifying uncertainty sources

1. **Confining pressure (σ₃)**
   a. Applied force
   b. Specimen base area
      i. Circumference diameter

2. **Deviation stress (σ₄)**
   a. Applied force
   b. Specimen base area
      i. Circumference diameter

3. **Number of Cycles (N)**

### 3.3. Determining the standard uncertainty for each source

The standard uncertainty corresponds to the uncertainty of a measurement denoted as a standard deviation (INMETRO, 2012) and can be classified as type A or type B. The statistical analysis of the first method refers to the values obtained by measurements, while the second encloses the other methods (ABNT and INMETRO, 2003).

In the case of the uncertainty’s sources outlined in Table 6, all must be considered type B, since they will be analyzed based on the manufacturer’s specifications, data from the calibration certificate and reference manuals.

#### 3.3.1. Confining pressure and deviation stress

Both confining pressure and deviation stress are defined by a force applied to an area. The force consists of the multiplication of the acceleration of gravity by the mass, measured, in the triaxial equipment, by a precision balance. The calibration certificate for this equipment states that there is an expanded uncertainty of 0.02 g and a coverage factor (k) of 2.0. From these data, the standard uncertainty of the equipment is calculated using Equation 4.

\[
u(\text{equipment}) = \frac{U_{dec}}{k_p}
\]  

where: \( u(\text{equipment}) \): equipment standard uncertainty; \( U_{dec} \): Expanded uncertainty stated by calibration certificate; and \( k_p \): coverage factor.

As for the area, it refers to the base of the specimen, whose diameter is measured by a pachymeter. Based on Carrasco, Carvalho and Oliveira (2008) work, the uncertainties related to the expanded uncertainty declared in the certificate (Equation 4) and to the resolution of the pachymeter (0.01 mm) were considered. It was assumed a triangular distribution for its uncertainty (Equation 5).

\[
u(\text{pachymeter}) = \frac{c}{\sqrt{b}}
\]  

where: \( c \): pachymeter resolution.

#### 3.3.2. Number of cycles

The duration of the load cycle (N) is defined by the adopted frequency. In the case of the samples tested, a frequency of 1 Hz was used, which means that 60 cycles per minute were applied.
Each cycle corresponds to 0.1 s of pulse, followed by a rest time of 0.9 s (DNIT, 2018).

During the test, this time is checked by an oscilloscope. According to the equipment certificate, the relative uncertainty varies between 1% and 3%. In this way, a uniform distribution (also called rectangular) is considered and the standard uncertainty is calculated according to Equation 6.

$$u(\text{oscilloscope}) = \frac{a_i}{\sqrt{3}}$$  \hspace{1cm} (6)

where: $a_i$: difference between the upper limit and the uniform distribution mean.

### 3.4. Determination of sensitivity coefficients $c_i$

The sensitivity coefficients represent the importance of each uncertainty source and are determined by the partial derivatives of $Y = f(X_1, X_2, ..., X_i, ..., X_n)$ regarding to $X_i$ at point $(x_1, x_2, ..., x_n)$. Therefore, each sensitivity coefficient is calculated by Equation 7.

$$c_i = \frac{\partial Y}{\partial X_i}$$  \hspace{1cm} (7)

where: $Y$: Function of the measurand, that is, $\varepsilon_p$; and $X_i$: Input parameters ($\sigma_s$, $\sigma_d$ ou N).

### 3.5. Combined standard uncertainty calculations

Once the sensitivity coefficient and standard uncertainty have been calculated, the uncertainty propagation, or error propagation, (Equation 8) is calculated to determine the combined standard uncertainty, represented by $u_c(y)$. However, since the input parameters are independent, Equation 8 can be simplified to Equation 9.

$$u_c(y) = \sqrt{\sum_{i=1}^{n} c_i^2 u_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_i c_j \text{cov}(x_i, x_j)}$$  \hspace{1cm} (8)

$$u_c(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)^2 (u_{x_i})^2}$$  \hspace{1cm} (9)

### 3.6. Expanded uncertainty calculation

To calculate the expanded uncertainty, the combined standard uncertainty must be multiplied by a coverage factor $k_p$ (Equation 10), determined from a Student distribution, with infinite degrees of freedom. This is because the data on the uncertainties of the measurands were not obtained by testing, which allows using the Gaussian distribution as a model, for which a coverage probability of 95% has a coverage factor $k_p=1.96$.

$$U = k_p \cdot u_c(y)$$  \hspace{1cm} (10)

where: $U$: Expanded uncertainty; $k_p$: Coverage factor for a coverage probability $p$; and $u_c(y)$: Combined standard uncertainty of the measurand.

### 3.7. Results expression

The permanent deformation result must be expressed according to Equation 11, in which the estimate of the measurand $Y$ is equivalent to the average of the measurements of the test.
\[ R_{med} = y \pm U \]  

where: \( R_{med} \): Measurement result [mm] 
\( y \): Estimate of the measurand, calculated in topic 3.1.3 [mm]; and 
\( U \): expanded uncertainty [mm]

According to ABNT and INMETRO (2003), this information must be accompanied by the measurement units of \( y \) and \( U \), and the \( k \) value used to obtain \( U \).

4. RESULTS

Basic sources standard uncertainties – force, diameter and number of cycles – and measurands sensitivity coefficient – area, stress and permanent deformation – were calculated, as shown in Tables 7 and 8, respectively.

<table>
<thead>
<tr>
<th>Table 7 – Input parameters standard uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Uncertainty source</strong></td>
</tr>
<tr>
<td>Force Certificate</td>
</tr>
<tr>
<td>Diameter Certificate</td>
</tr>
<tr>
<td>Resolution</td>
</tr>
<tr>
<td>Number of cycles Certificate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8 – Measurands sensitivity coefficient calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurand: Area ( A = (\pi \times (\Phi/2))^2 )</strong></td>
</tr>
<tr>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>( \Phi )</td>
</tr>
</tbody>
</table>

| **Measurand: Stress = \( \sigma = F/A \)** |
| **Symbol** | Description | Partial derivative | Unit | \( c_1 \) estimate (G1) | \( c_1 \) estimate (G2) |
| \( F \) | Force | \( \partial \sigma / \partial F = 1/A \) | 1/cm² | 0.012732395 | 0.012732395 |
| \( A \) | Area | \( \partial \sigma / \partial A = -F/A^2 \) | -kgf/cm⁴ | -0.00909457 | -0.0272837 |

| **Measurand: Permanent Deformation \( \varepsilon_p = \psi_1 \left( \frac{\sigma_3}{\rho_b} \right)^{(V_3 - 1)} \left( \frac{\sigma_d}{\rho_d} \right)^{V_4} N^{V_2} \)** |
| **Symbol** | Description | Partial derivative | Unit | \( c_1 \) estimate (G1) | \( c_1 \) estimate (G2) |
| \( \sigma_3 \) | Confining pressure | \( \partial \varepsilon_p / \partial \sigma_3 = \psi_1 \psi_2 \left( \frac{\sigma_3}{\rho_b} \right)^{(V_3 - 1)} \left( \frac{\sigma_d}{\rho_d} \right)^{V_4} N^{V_2} \) | cm²/kgf | 0.085273715 | 0.483276907 |
| \( \sigma_d \) | Deviation stress | \( \partial \varepsilon_p / \partial \sigma_d = \psi_1 \psi_2 \left( \frac{\sigma_3}{\rho_b} \right)^{(V_3 - 1)} \left( \frac{\sigma_d}{\rho_d} \right)^{V_4} N^{V_2} \) | cm²/kgf | 1.447819306 | 2.735079263 |
| \( N \) | Number of cycles | \( \partial \varepsilon_p / \partial N = \psi_1 \psi_2 \left( \frac{\sigma_3}{\rho_b} \right)^{(V_3 - 1)} \left( \frac{\sigma_d}{\rho_d} \right)^{V_4} N^{(V_2 - 1)} \) | adm. | 7.20433×10⁷ | 4.08295×10⁶ |

It is shown in Table 9 the standard and combined uncertainties calculation. Diameter, area and number of cycles are common measurements to both groups analyzed in this work. Force 1 and Force 2 are considered to be those necessary to apply, in a circle of 10 cm diameter (referring to the specimen used in the tests), Stresses A and B, respectively.

Necessary data to calculate expanded uncertainties of both analyzed groups are presented in Table 10. It is observed that Stress A corresponds to the confining pressure applied to G1 and G2 groups and to the deviation stress applied only to G1 group. Stress B refers to the deviation stress applied to G2 group only.

Measurand estimate for G1 group is 1.589 mm, and 5.981 mm for G2 group. Thus, the measurements uncertainty in the permanent deformation tests are expressed as follows:

- For G1 – Permanent deformation expressed as \((1.589 \pm 0.062)\) mm, \(k = 1.96\)
- For G2 – Permanent deformation expressed as \((5.981 \pm 0.353)\) mm, \(k = 1.96\)
Table 9 – Standard and combined uncertainties calculation

<table>
<thead>
<tr>
<th>Measurand:</th>
<th>Force 1 = 56.10 kgf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Data</td>
<td>Value</td>
</tr>
<tr>
<td>Certificate</td>
<td>0.00002</td>
</tr>
<tr>
<td>Measurand:</td>
<td>Force 2 = 168.30 kgf</td>
</tr>
<tr>
<td>Uncertainty Data</td>
<td>Value</td>
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<tr>
<td>Certificate</td>
<td>0.00002</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Measurand:</th>
<th>Diameter = 100 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Data</td>
<td>Value</td>
</tr>
<tr>
<td>Certificate</td>
<td>0.013</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Measurand: | Area = π x (D/2)² = 78.54 cm² |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Data</td>
<td>Value</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.00768</td>
</tr>
</tbody>
</table>

Measurand: | Stress A = Force / Area = 0.7143 kgf/cm² |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (certific.)</td>
<td>0.00001</td>
</tr>
<tr>
<td>Area (cert/resol)</td>
<td>0.01206</td>
</tr>
</tbody>
</table>

Measurand: | Stress B = Force / Area = 2.1428 kgf/cm² |
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Force (certific.)</td>
<td>0.00001</td>
</tr>
<tr>
<td>Area (cert/resol)</td>
<td>0.01206</td>
</tr>
</tbody>
</table>

Measurand: | Number of cycles = 50,000 |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Certificate</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 10 – G1 and G2 groups expanded uncertainty calculation

<table>
<thead>
<tr>
<th>Measurand:</th>
<th>Permanent deformation G1 = 1.589 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Data</td>
<td>Value</td>
</tr>
<tr>
<td>Stress σdA (certific.)</td>
<td>0.00011</td>
</tr>
<tr>
<td>Stress σdB (certific.)</td>
<td>0.00011</td>
</tr>
<tr>
<td>Number of cycles (certific.)</td>
<td>0.00577</td>
</tr>
</tbody>
</table>

Combined uncertainty - εp G1 (h specimen = 200 mm) | 0.03181 | mm |

k p for 95% coverage probability: | 1.96 |

Expanded uncertainty | 0.06234 | mm |

5. DISCUSSIONS

It was observed that the most influential parameter for measurement uncertainty in both groups is stress. The 300% increase in σd between G1 and G2 was responsible for an increase in the combined uncertainty of more than 560% between the groups.
Number of Cycles has the highest standard uncertainty value; however, the sensitivity coefficient value (around $2 \times 10^{-6}$) reduces significantly this factor contribution to combined uncertainty.

It was observed that the first group presented lower measurement uncertainties values, equivalent to 3.9% of mean permanent deformation. For the second group, whose permanent deformations were considerably higher, the same parameter reached 5.9%. Therefore, the increase in the deviation stress were responsible for higher values of measurement uncertainty.

In order to assess estimating measurement uncertainties relevance for road pavements, in this study, data obtained from pavement models found in the literature were used. Lima, Motta and Aragão (2019) proposed models of flexible road pavements, consisting of clayey soil subgrade (NG'), 30 cm lateritic clayey soil (LG') subbase, 8 cm asphalt coating and 30 cm base with variable soils (NS', NG' or NA').

The combination that presented higher rutting, 6.31 mm, was the NS' classification base, which, presented individually a 1.87 mm permanent deformation. This value is similar to the one found for the G1 group lateritic clayey soil (1.59 mm).

Replacing this pavement base with the soil presented in this work, it can be observed that, determining the uncertainty expanded only for the layer, the total permanent deformation would be 6.09 mm. It is noted that the use of G2 group soil would be unfeasible, regardless of the result of the expanded uncertainty estimate, due to its high deformation.

Bezerra, Gonzaga and Oliveira (2020) analyzed a model of rigid pavement with a 5 cm settlement layer; 20 cm base, 22 cm sub-base and subgrade. Among the eight different scenarios proposed by the authors, the one that presented the highest value of permanent deformation was scenario 5, with a 4.0629 cm wheel track. In this context, the base presented a 3.4181 cm PD. By replacing this base material with the soil of the G2 group, which has higher values, the permanent deformation became 6.9788 cm, including 0.353 cm of the measurement uncertainty of the group.

Regarding road pavement non-compliance standards, the maximum permissible value for permanent deformation adopted by the Federal Aviation Administration is 12.7 mm (FAA, 2014), a higher value than the one adopted by Belgium, where rutting can reach 16 mm (Santos, 1998). In Brazil, ARTESP (2014) uses stricter criteria, establishing a maximum value of 7 mm for wheel track.

It was observed that, for the analyzed scenarios, the pavements did not present permanent deformation higher than 7 mm, even after determined measurement uncertainties estimates. Only in the case where G2 group was used, in the flexible pavement, the DP would be higher than the maximum value, however, due to the high deformability of the soil, regardless of the result of the uncertainty estimates.

6. CONCLUSION

This article analyzed measurement uncertainties estimates based on tests carried out by Guimarães (2009), and the interval of more than a decade between the data collecting and this study has become a challenge for obtaining information, such as the uncertainty-standard of equipment used.

Due to the absence of a calibration certificate for the repeated loads triaxial equipment, the data referring to the equipment associated with each of the uncertainty sources were used as
standard uncertainties estimates; in the absence of the equipment calibration certificates, data informed in the respective manuals were adopted.

From the evaluation of pavement models found in the literature, it was pointed that, for the studied soil, the estimates of measurement uncertainties guaranteed the quality of the results, because every measurement has an error associated with it, and without a quantitative determination of this error, the measurement has no value.

It should be noted that the analysis was performed considering the measurement uncertainty of only one of the layers, which is a limitation of the work. The inclusion of uncertainties associated with the other layers should be part of a study to verify the influences on the result of the total deformation of the pavement.

It is also suggested the analysis of the influence of other parameters as sources of uncertainty: confining pressure, number of cycles and moisture content, from the isolated variation of the source; analysis in coarse materials, repetition of the procedure at different cycle values, and determination of the estimate of PD measurement uncertainties, obtained through other methods, such as RLT.

This work is limited to the evaluation of the presented soil measurement uncertainty, and other samples with different classifications and properties may present results that could make the material application unfeasible in pavement layers.

It is concluded that, when reporting the measurement result of a quantity, it is essential that there is a quantitative indication of the quality of the result, in such a way that those who use it can assess its reliability. Without this indication, measurement results cannot be compared, either with each other or with reference values given in the specification or standards. It is therefore necessary that there is an implemented, easily understood and generally accepted procedure to characterize the quality of a measurement result through its uncertainty.

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